Problem 2.2

The origin of the linear drag force on a sphere in a fluid is the viscosity of the fluid. According to Stokes's law, the viscous drag on a sphere is

$$f_{\text{lin}} = 3\pi\eta Dv \tag{2.82}$$

where η is the viscosity[8] of the fluid, D the sphere's diameter, and v its speed. Show that this expression reproduces the form (2.3) for f_{lin}, with b given by (2.4) as $b = \beta D$. Given that the viscosity of air at STP is $\eta = 1.7 \times 10^{-5}\ \text{N}\cdot\text{s/m}^2$, verify the value of β given in (2.5).

Solution

The aim is to show that the given expression for f_{lin} simplifies to

$$f_{\text{lin}} = bv, \tag{2.3}$$

where $b = \beta D$ for spherical projectiles and $\beta = 1.6 \times 10^{-4}\ \text{N}\cdot\text{s/m}^2$ for projectiles in air at STP.

$$\begin{aligned}
f_{\text{lin}} &= 3\pi\eta Dv \\
&= \underbrace{(3\pi\eta)}_{=\beta} Dv \\
&= \beta Dv \\
&= \underbrace{(\beta D)}_{=b} v \\
&= bv
\end{aligned}$$

Check to see that this relationship between η and β yields the right value for β.

$$\begin{aligned}
\beta &\stackrel{?}{=} 3\pi\eta \\
&\stackrel{?}{=} 3\pi\left(1.7 \times 10^{-5}\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}\right) \\
&\approx 1.6 \times 10^{-4}\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}
\end{aligned}$$

It checks out.

[8] For the record, the viscosity η of a fluid is defined as follows: Imagine a wide channel along which fluid is flowing (x direction) such that the velocity v is zero at the bottom ($y = 0$) and increases toward the top ($y = h$), so that successive layers of fluid slide across one another with a velocity gradient dv/dy. The force F with which an area A of any one layer drags the fluid above it is proportional to A and to dv/dy, and η is defined as the constant of proportionality; that is, $F = \eta\, A\, dv/dy$.